PRACTICAL ASSIGNMENT

MSMS – 407 : Practical based on above papers

Submitted by

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Practical Assignment

Problem 1

Generate 100 observations from N (0,1) using the Box-Muller transformation method and Acceptance Rejection method. You can use double exponential distribution as proposal density. Hence generate from 100 observations from N (2, 10)

1.Box-Muller Method

Algorithm

1. Generate two random variables u_1 and u_2 from Standard Uniform Distribution
2. Then set

$$Z_1 = \sqrt{-2log(U_1)}cos(2\pi U_2)$$

$$Z_2 = \sqrt{-2log(U_1)}sin(2\pi U_2)$$

3. Now X_1 and X_2 are independent standard normal variates , now we can transform using following equation

$$Y_1 = X_1\sigma + \mu$$

$$Y_2 = X_2\sigma + \mu$$

Now Y_1 and Y_2 are distributed

$$Y_1, Y_2 \sim N(\mu, \sigma^2)$$

Code

```
# To Genereate Random numbers from standard uniform distribution we will us runif() function

U_1 <- runif(50)
U_2 <- runif(50)

# Now Converting Uniform to Standard normal using Box-Muller Method

Z_1 = sqrt(-2*log(U_1))*cos(2*pi*U_2)
Z_2 = sqrt(-2*log(U_2))*sin(2*pi*U_1)

Z <- c(Z_1,Z_2)
print(Z)
```

```
##   [1] -0.17379567  0.68153044  0.43588063  0.32639272 -1.00962265 -1.06439258
##   [7] -0.24345821 -0.62187939 -1.19055170  1.12901894  1.50217148 -0.55009361
##  [13] -0.76149107  2.21154299 -0.01079774  1.38087549  1.18475082  0.42274538
##  [19]  1.03283520  0.99816181  0.04439397  0.52985986 -0.58530514 -0.06136478
##  [25] -1.29188693 -0.63619458  0.21185564 -0.10411944 -0.22787286  0.47086157
##  [31] -0.44569445 -0.80387781  1.11368075 -0.48170133 -0.68497063 -1.07467222
```

```
## [37] -2.05132605  0.90546239 -1.00951453  0.71255726  0.29771583  0.05540006
## [43] -1.21881772 -0.64955309 -1.06157260 -0.68202672 -2.00753672 -1.18222646
## [49]  0.70129659 -1.23461662 -1.07649607 -0.22391495 -1.82264651 -0.62638411
## [55] -0.69140744  0.84214576 -0.30567590 -1.02795240  0.07796159  0.46449429
## [61]  2.12044068 -0.89182777 -1.18606230  1.71360022 -0.94930350  0.87375749
## [67]  0.01069264 -1.44211639  1.53383090  1.40233359 -0.59098378  0.46076603
## [73]  1.30138782 -0.33640842  0.47283311 -1.05083749 -0.03589455 -1.60485058
## [79] -0.55953725 -0.10619766 -0.83613957 -1.17602396 -0.19388490  0.77201646
## [85]  1.51358999 -0.36334577  0.36259299  1.39144813 -0.28042191  0.62895265
## [91]  0.70724738 -1.68974485  1.37915362 -1.10187206  1.42027393 -1.13343633
## [97]  0.87251924  0.22864644  1.19651017  0.87932572
```

To transform these to $N(2, 10)$ we will us

$$X = Z_{0,1}\sigma + \mu$$

So we have $\mu = 2$ and $\sigma = \sqrt{10}$ so

```
X = Z*sqrt(10)+2
print(X)
```

```
##   [1]  1.45040982  4.15518847  3.37837556  3.03214441 -1.19270716 -1.36590487
##   [7]  1.23011755  0.03344469 -1.76485504  5.57027138  6.75028332  0.26045128
##  [13] -0.40804621  8.99351301  1.96585454  6.36671170  5.74651105  3.33683827
##  [19]  5.26611169  5.15646478  2.14038607  3.67556400  0.14910262  1.80594754
##  [25] -2.08530518 -0.01182392  2.66994637  1.67074541  1.27940276  3.48899503
##  [31]  0.59059041 -0.54208483  5.52176775  0.47672665 -0.16606732 -1.39841195
##  [37] -4.48686254  4.86332348 -1.19236525  4.25330391  2.94146011  2.17519038
##  [43] -1.85424005 -0.05406722 -1.35698732 -0.15675788 -4.34838853 -1.73852831
##  [49]  4.21769455 -1.90420054 -1.40417947  1.29191875 -3.76371435  0.01919952
##  [55] -0.18642230  4.66309872  1.03336794 -1.25067092  2.24653619  3.46885990
##  [61]  8.70542219 -0.82020703 -1.75065831  7.41887969 -1.00196126  4.76306378
##  [67]  2.03381309 -2.56037245  6.85039919  6.43456818  0.13114520  3.45707013
##  [73]  6.11534961  0.93618318  3.49522957 -1.32303993  1.88649145 -3.07498314
##  [79]  0.23058787  1.66417351 -0.64410549 -1.71891431  1.38688212  4.44133042
##  [85]  6.78639181  0.85099979  3.14661970  6.40014535  1.11322805  3.98892292
##  [91]  4.23651259 -3.34344239  6.36126669 -1.48442539  6.49130054 -1.58424038
##  [97]  4.75914809  2.72304354  5.78369739  4.78067209
```

2. Acceptance Rejection Method

Algorithm

1. Choose a density that is easy to sample from.
2. Find a constant c such that Equation 4.6 is satisfied.
3. Generate a random number Y from the density .
4. Generate a uniform random number U.
5. If

$$U \leq \frac{f(Y)}{cg(Y)}$$

   , then accept X=Y , else go to step 3

First of all we will solve following inequality

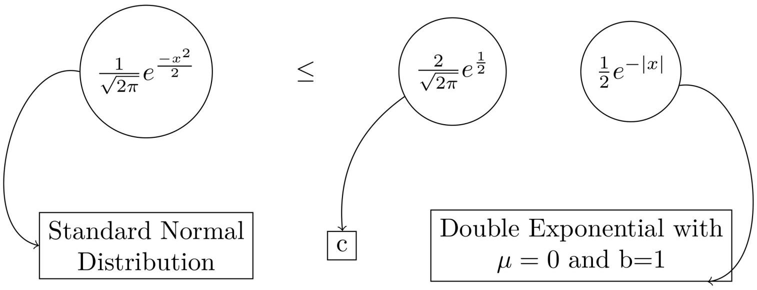
$$\frac{1}{2}(|x| - 1)^2 \geq 0$$

$$\Rightarrow \frac{1}{2}(x^2 + 1 - 2|x|) \geq 0$$

$$\Rightarrow \frac{1}{2}x^2 + \frac{1}{2} - |x| \geq 0$$

$$\frac{-x^2}{2} \leq \frac{1}{2} - |x|$$

$$\frac{1}{\sqrt{2\pi}} e^{\frac{-x^2}{2}} \leq \frac{1}{\sqrt{2\pi}} e^{\frac{1}{2}-|x|}$$



Now we have c = 1.3154892 so now

Code

```
c = (2*exp(0.5))/sqrt(2*pi)
Z = c()
while(length(Z) < 100){
  a = rlaplace(1)
  u = runif(1)
  if( u <= dnorm(a)/(c*dlaplace(a))){
    Z = c(Z,a)}
}
print(Z)
```

```
##   [1]  0.50379376  0.84718517 -0.31652393 -0.99849262  1.15232095  0.78807060
##   [7]  0.14717076 -0.85559646  1.43827133 -0.92068039  1.22377419 -1.00383856
##  [13]  0.33665475 -0.51385016 -0.95029580  1.21475944  1.11830397  0.41569651
##  [19]  0.32193055 -0.49757534  0.84900648 -0.10950972  0.44713618  0.57728493
##  [25]  0.18828238 -0.04683103  0.25098764  0.12338719 -0.61898634  0.99606023
##  [31] -1.29365759 -0.94807143  0.32394370  0.91542255  1.27587766 -2.04767306
##  [37]  0.36399468  0.36345637  0.77940609  0.61272328  0.08049606  0.45775524
##  [43]  0.69243113 -0.64834320  0.49843780  0.39880048 -1.31355335  1.53824654
##  [49]  0.80495250  0.64106658  0.27927370 -0.73764418  0.99808747  0.65853811
##  [55] -0.03859097 -1.60994265  0.41889010 -1.14248590 -0.26977847 -0.16885982
##  [61] -0.83385940  0.50997719 -0.07478001 -0.07273755 -2.04370635  1.14305461
##  [67] -2.13284945 -0.46071695 -0.41564908  1.25023490 -0.10597037  0.43862393
##  [73] -0.61868603 -0.04173852 -0.35211231 -0.89058099 -0.77220789 -0.54206509
##  [79]  0.71056921  0.33023729 -0.60321162  0.75821441 -0.80445072  0.96333834
##  [85] -1.27913131 -0.28171597 -1.29368102 -2.06433598 -0.28105272 -0.69700992
##  [91]  2.19454066  1.76133052  0.30665610  0.09626922 -0.32757525 -0.32352353
##  [97] -0.34589649 -1.77448604 -0.08132519  0.31856421
```

So we have $\mu = 2$ and $\sigma = \sqrt{10}$ so

```
X = Z*sqrt(10)+2
print(X)
```

```
##   [1]  3.59313574  4.67903472  0.99906344 -1.15751089  5.64395880  4.49209805
##   [7]  2.46539480 -0.70563356  6.54821330 -0.91144703  5.86991379 -1.17441627
##  [13]  3.06459580  0.37506312 -1.00509917  5.84140663  5.53638768  3.31454778
```

```
## [19]  3.01803379  0.42652861  4.68479422  1.65369984  3.41396874  3.82553524
## [25]  2.59540117  1.85190728  2.79369261  2.39018455  0.04259334  5.14981901
## [31] -2.09090451 -0.99806509  3.02439992  4.89482027  6.03467944 -4.47531079
## [37]  3.15105225  3.14934997  4.46469848  3.93760115  2.25455090  3.44754918
## [43]  4.18965948 -0.05024121  3.57619872  3.26111786 -2.15382041  6.86436268
## [49]  4.54548331  4.02723053  2.88314100 -0.33263573  5.15622971  4.08248036
## [55]  1.87796463 -3.09108567  3.32464680 -1.61285762  1.14688559  1.46601837
## [61] -0.63689496  3.61268947  1.76352485  1.76998366 -4.46276692  5.61465606
## [67] -4.74466217  0.54308510  0.68560220  5.95358990  1.66489227  3.38705064
## [73]  0.04354299  1.86801122  0.88652312 -0.81626437 -0.44193577  0.28583969
## [79]  4.24701713  3.04430199  0.09247738  4.39768448 -0.54389655  5.04634332
## [85] -2.04496836  1.10913589 -2.09097858 -4.52800354  1.11123326 -0.20413891
## [91]  8.93974690  7.56981615  2.96973173  2.30442999  0.96411612  0.97692876
## [97]  0.90617924 -3.61141757  1.74282716  3.00738850
```

Problem 2

Let us consider the following dataset follows an exponential distribution with scale parameter θ.Let us consider the prior for θ. Obtain posterior distribution, Bayes estimator, and 0.95 HPD interval for the parameter.

3.29, 7.53, 0.48, 2.03, 0.36, 0.07, 4.49, 1.05, 9.15,3.67, 2.22, 2.16, 4.06, 11.62, 8.26, 1.96, 9.13, 1.78, 3.81, 17.02

The density of the data model will be given by

$$f(x|\theta) = \frac{1}{\theta} e^{\frac{-x}{\theta}}$$

Let us notify $\sum_{i=1}^{n} x_i = S_n$ now the likelihood will be given by

$$L(x|\theta) = \left(\frac{1}{\theta}\right)^n e^{\frac{-S_n}{\theta}}$$

Now Since we do not have any info about θ let us assume non-informative prior

$$\pi(\theta) = \frac{1}{\theta}$$

Then the posterior will be given by

$$\pi(\theta|x) = \frac{\frac{1}{\theta} \cdot \left(\frac{1}{\theta}\right)^n e^{\frac{-S_n}{\theta}}}{\int_0^{\infty} \frac{1}{\theta} \cdot \left(\frac{1}{\theta}\right)^n e^{\frac{-S_n}{\theta}}}$$

$$\pi(\theta|x) = \frac{S_n^n}{\Gamma(n)} \cdot \left(\frac{1}{\theta}\right)^{n+1} e^{\frac{-S_n}{\theta}}$$

Now this is the density of the Inverse Gamma so

$$\pi(\theta|x) \sim Inv - Gamma(n, S_n)$$

So the bayes estimate will be given by $\frac{S_n}{n-1}$

Code

```r
xobs <- c(3.29, 7.53, 0.48, 2.03, 0.36, 0.07, 4.49, 1.05, 9.15,3.67, 2.22, 2.16, 4.06, 11.62, 8.26, 1.9
Bayes_Estimate = sum(xobs)/(length(xobs)-1)
cat("Bayes Estimate of scale parameter is given by ",Bayes_Estimate)
```

```
## Bayes Estimate of scale parameter is given by  4.954737
```

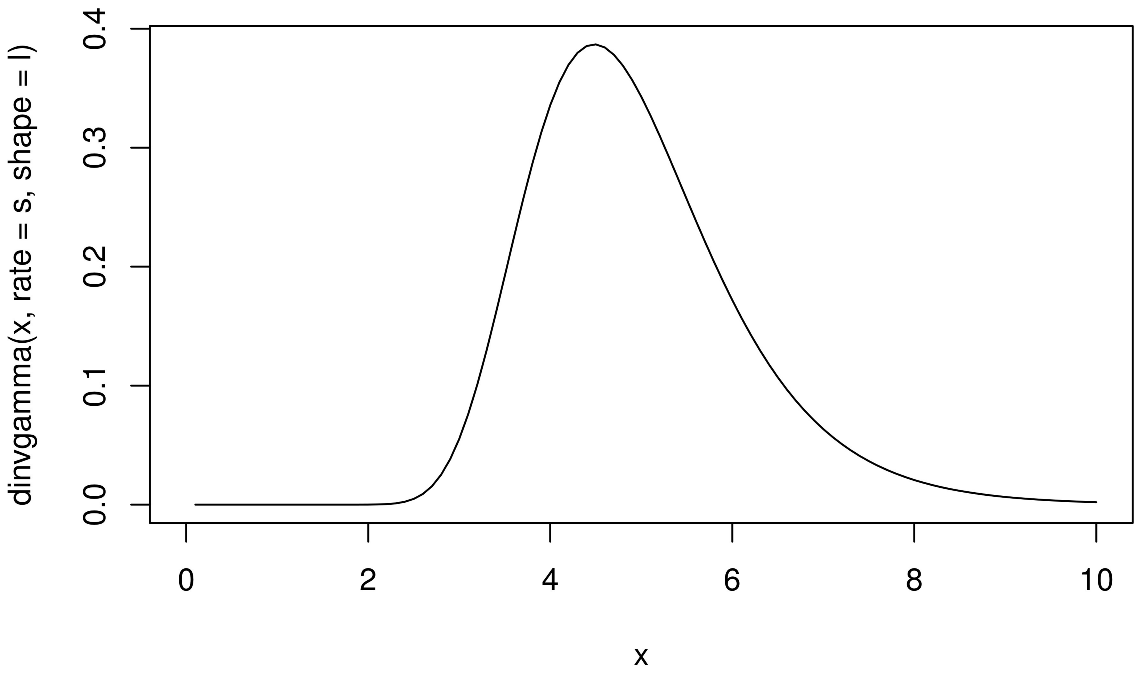
Now **HPDI** will br given by

$$\int_{\theta:\pi(\theta|X)\geq k} \pi(\theta|X)d\theta = 1-\alpha$$

where $1-\alpha = 0.95$, here it can be thought as a horizontal line is on the posterior density such that the point where the posterior density intersect this line the area between these points will be 0.95

Let us take a look at posterior density function

```
s = sum(xobs)
l =length(xobs)
curve(dinvgamma(x , rate = s , shape = l),from=0,to=10)
```



Now let us find HPD **Code**

```
ruler1 <- seq(2, s/(l+1),length=3500 )  #s\(l+1) is mode of posterior
ruler2 <- seq(s/(l+1), 8 ,length = 5000)
target = 0.95
tolerance = 0.0005
done<- FALSE
for(i in ruler1)
{
  for(j in ruler2)
  {
    if(round(dinvgamma(i,rate=s,shape = l),3)==round(dinvgamma(j,rate=s,shape = l),3))
```

```r
    {
      #print(paste(i,"and",j))
      L <- pinvgamma(i,rate=s,shape=l)
      H <- pinvgamma(j,rate=s,shape=l)
      if (((H-L)<(target+tolerance)) & ((H-L)>(target-tolerance)))
      {
        done <- TRUE
        break
      }
    }
  }
 if (done){break}
}
HPD.L <- i; HPD.U <- j
print(paste(target*100, "% HPD interval:", HPD.L, "to", HPD.U))
```

```
## [1] "95 % HPD interval: 2.94588413015964 to 7.2851736061498"
```