Bisection Methos in R

Rahul Goswami

2022-01-26

Bisection Method is a method for finding the root of a function. Suppose we want to find the root of the equation

$$f(x) = 0$$

in the interval $[a, b]$. Steps for the Bisection Method are:

1. Take the midpoint of the interval $[a, b]$, $x_0 = (a + b)/2$
2. Evaluate the function at x_0 , $f(x_0)$
3. If $f(x_0) = 0$ then the root is found.
4. If sign of $f(x_0)$ is same as sign of $f(a)$ then $a = x_0$ and $b = b$ otherwise $a = a$ and $b = x_0$
5. Repeat the process until the interval converges to a root.

Example

f(x) = x^2 + 4x - 7 in the interval $[-10, 0]$ f(-10) = -10^2 + 4*(-10) - 7 = -70 and f(0) = 0 Opposite sign of f(-10) and f(0) so we will start with $a = -10$ and $b = 0$

```
f <- function(x) {
  x^2 + 4*x - 7
}
```

Initial points

```
a <- -10
b <- 0
count <- 0
midpoint <- c()
functionvalue <- c()
while(abs(a-b) > 0.00001) {
  count = count + 1
  x <- (a+b)/2  # midpoint
  midpoint <- c(midpoint,x) # store the midpoint
  fx <- f(x)    # evaluate f(x) at midpoint
  functionvalue <- c(functionvalue,fx) # store the function value
  if(sign(f(a)) == sign(fx)) {
    a <- x
  } else {
    b <- x
  }
  cat("Iteration:",count,"\t","f(x) =",fx,"midpoint:",x,"New interval:",a,"to",b,"\n")
}
```

```
## Iteration: 1      f(x) = -2 midpoint: -5 New interval: -10 to -5
## Iteration: 2      f(x) = 19.25 midpoint: -7.5 New interval: -7.5 to -5
## Iteration: 3      f(x) = 7.0625 midpoint: -6.25 New interval: -6.25 to -5
## Iteration: 4      f(x) = 2.140625 midpoint: -5.625 New interval: -5.625 to -5
```

```
## Iteration: 5      f(x) = -0.02734375 midpoint: -5.3125 New interval: -5.625 to -5.3125
## Iteration: 6      f(x) = 1.032227 midpoint: -5.46875 New interval: -5.46875 to -5.3125
## Iteration: 7      f(x) = 0.4963379 midpoint: -5.390625 New interval: -5.390625 to -5.3125
## Iteration: 8      f(x) = 0.2329712 midpoint: -5.351562 New interval: -5.351562 to -5.3125
## Iteration: 9      f(x) = 0.1024323 midpoint: -5.332031 New interval: -5.332031 to -5.3125
## Iteration: 10     f(x) = 0.03744888 midpoint: -5.322266 New interval: -5.322266 to -5.3125
## Iteration: 11     f(x) = 0.005028725 midpoint: -5.317383 New interval: -5.317383 to -5.3125
## Iteration: 12     f(x) = -0.01116347 midpoint: -5.314941 New interval: -5.317383 to -5.314941
## Iteration: 13     f(x) = -0.003068864 midpoint: -5.316162 New interval: -5.317383 to -5.316162
## Iteration: 14     f(x) = 0.0009795576 midpoint: -5.316772 New interval: -5.316772 to -5.316162
## Iteration: 15     f(x) = -0.001044746 midpoint: -5.316467 New interval: -5.316772 to -5.316467
## Iteration: 16     f(x) = -3.261771e-05 midpoint: -5.31662 New interval: -5.316772 to -5.31662
## Iteration: 17     f(x) = 0.0004734641 midpoint: -5.316696 New interval: -5.316696 to -5.31662
## Iteration: 18     f(x) = 0.0002204218 midpoint: -5.316658 New interval: -5.316658 to -5.31662
## Iteration: 19     f(x) = 9.390166e-05 midpoint: -5.316639 New interval: -5.316639 to -5.31662
## Iteration: 20     f(x) = 3.064188e-05 midpoint: -5.316629 New interval: -5.316629 to -5.31662
```

Plottings

```
{plot(f,xlim=c(-10,0),ylim=c(-13,7),xlab="x",ylab="f(x)",main="Bisection Method")
points(midpoint,functionvalue,pch=19,col="red",cex=0.5)
lines(c(0,-10),c(0,0),col="blue",lwd=0.5)}
```