

Ridge Regression in R

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Ridge Regression is a statistical method for estimating the parameters of a linear model, where the independent variables are highly correlated, It is a way to reduce the variance of the estimates by imposing a penalty on the absolute value of the coefficients. the penalty decides the amount of regularization, In ridge penalty is added to the sum of squares of the coefficients In genral we try to reduce

$$RSS = \sum_{i=1}^n (y_i - \hat{y}_i)^2$$

but in Ridge Regression we add a penalty to the sum of squares of the coefficients

$$\sum_{i=1}^n (y_i - \hat{y}_i)^2 + \lambda \sum_{j=1}^p b_j^2$$

where λ is the penalty parameter, and the whole second term is known as Shrinkage.

Ridge Regression

$$b = (X^T X + \lambda I)^{-1} X^T y$$

Dataset

```
nobs <- 100 # number of observations
ntrain <- 0.8*nobs # number of training observations
x1 <- runif(nobs) # independent variable
x2 <- x1 + rnorm(nobs,0,0.2) # independent variable
y <- 4 + 3*x1 + 2*x2 + rnorm(nobs) # dependent variable
X = matrix(c(rep(1,nobs),x1,x2),ncol=3) # design matrix
train.X <- X[1:ntrain,] # training design matrix
train.y <- y[1:ntrain] # training dependent variable
test.X <- X[(ntrain+1):nobs,] # test design matrix
test.y <- y[(ntrain+1):nobs] # test dependent variable

for(lambda in seq(0,4,0.1)){
  #' Ridge Regression
  b = solve(t(train.X) %*% train.X + lambda*diag(1,3,3), t(train.X) %*% train.y)
  #' b using OLS
  b.ols <- solve(t(train.X) %*% train.X, t(train.X) %*% train.y)
  #' Prediction
  pred.y <- test.X %*% b
  #' Prediction using OLS
  pred.ols <- test.X %*% b.ols
  #' Residuals
  resid.y <- test.y - pred.y
  #' Residuals using OLS
  resid.ols <- test.y - pred.ols
}
```

```

#' RSS
RSS <- sum(resid.y^2)
#' RSS using OLS
RSS.ols <- sum(resid.ols^2)
#' R-squared
R2 <- 1 - RSS/sum((test.y - mean(test.y))^2)
#' R-squared using OLS
R2.ols <- 1 - RSS.ols/sum((test.y - mean(test.y))^2)
cat("lambda = ",lambda," , RSS = ",RSS," , R2 = ",R2,"\n")
cat("lambda = ",lambda," , RSS = ",RSS.ols," , R2 = ",R2.ols,"\n", "\n")
}

```

```

## lambda = 0 , RSS = 32.53004 , R2 = 0.6148474
## lambda = 0 , RSS = 32.53004 , R2 = 0.6148474
##
## lambda = 0.1 , RSS = 32.5194 , R2 = 0.6149733
## lambda = 0.1 , RSS = 32.53004 , R2 = 0.6148474
##
## lambda = 0.2 , RSS = 32.51044 , R2 = 0.6150794
## lambda = 0.2 , RSS = 32.53004 , R2 = 0.6148474
##
## lambda = 0.3 , RSS = 32.50256 , R2 = 0.6151728
## lambda = 0.3 , RSS = 32.53004 , R2 = 0.6148474
##
## lambda = 0.4 , RSS = 32.49537 , R2 = 0.6152579
## lambda = 0.4 , RSS = 32.53004 , R2 = 0.6148474
##
## lambda = 0.5 , RSS = 32.48867 , R2 = 0.6153372
## lambda = 0.5 , RSS = 32.53004 , R2 = 0.6148474
##
## lambda = 0.6 , RSS = 32.48234 , R2 = 0.6154122
## lambda = 0.6 , RSS = 32.53004 , R2 = 0.6148474
##
## lambda = 0.7 , RSS = 32.47631 , R2 = 0.6154835
## lambda = 0.7 , RSS = 32.53004 , R2 = 0.6148474
##
## lambda = 0.8 , RSS = 32.47057 , R2 = 0.6155515
## lambda = 0.8 , RSS = 32.53004 , R2 = 0.6148474
##
## lambda = 0.9 , RSS = 32.46512 , R2 = 0.6156161
## lambda = 0.9 , RSS = 32.53004 , R2 = 0.6148474
##
## lambda = 1 , RSS = 32.45998 , R2 = 0.615677
## lambda = 1 , RSS = 32.53004 , R2 = 0.6148474
##
## lambda = 1.1 , RSS = 32.45518 , R2 = 0.6157338
## lambda = 1.1 , RSS = 32.53004 , R2 = 0.6148474
##
## lambda = 1.2 , RSS = 32.45076 , R2 = 0.6157861
## lambda = 1.2 , RSS = 32.53004 , R2 = 0.6148474
##
## lambda = 1.3 , RSS = 32.44675 , R2 = 0.6158336
## lambda = 1.3 , RSS = 32.53004 , R2 = 0.6148474
##

```

```
## lambda = 1.4 , RSS = 32.4432 , R2 = 0.6158756
## lambda = 1.4 , RSS = 32.53004 , R2 = 0.6148474
##
## lambda = 1.5 , RSS = 32.44014 , R2 = 0.6159118
## lambda = 1.5 , RSS = 32.53004 , R2 = 0.6148474
##
## lambda = 1.6 , RSS = 32.43762 , R2 = 0.6159417
## lambda = 1.6 , RSS = 32.53004 , R2 = 0.6148474
##
## lambda = 1.7 , RSS = 32.43566 , R2 = 0.6159649
## lambda = 1.7 , RSS = 32.53004 , R2 = 0.6148474
##
## lambda = 1.8 , RSS = 32.43429 , R2 = 0.6159811
## lambda = 1.8 , RSS = 32.53004 , R2 = 0.6148474
##
## lambda = 1.9 , RSS = 32.43356 , R2 = 0.6159898
## lambda = 1.9 , RSS = 32.53004 , R2 = 0.6148474
##
## lambda = 2 , RSS = 32.43348 , R2 = 0.6159907
## lambda = 2 , RSS = 32.53004 , R2 = 0.6148474
##
## lambda = 2.1 , RSS = 32.43409 , R2 = 0.6159835
## lambda = 2.1 , RSS = 32.53004 , R2 = 0.6148474
##
## lambda = 2.2 , RSS = 32.4354 , R2 = 0.6159679
## lambda = 2.2 , RSS = 32.53004 , R2 = 0.6148474
##
## lambda = 2.3 , RSS = 32.43745 , R2 = 0.6159437
## lambda = 2.3 , RSS = 32.53004 , R2 = 0.6148474
##
## lambda = 2.4 , RSS = 32.44025 , R2 = 0.6159106
## lambda = 2.4 , RSS = 32.53004 , R2 = 0.6148474
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## lambda = 2.5 , RSS = 32.44381 , R2 = 0.6158683
## lambda = 2.5 , RSS = 32.53004 , R2 = 0.6148474
##
## lambda = 2.6 , RSS = 32.44817 , R2 = 0.6158168
## lambda = 2.6 , RSS = 32.53004 , R2 = 0.6148474
##
## lambda = 2.7 , RSS = 32.45333 , R2 = 0.6157557
## lambda = 2.7 , RSS = 32.53004 , R2 = 0.6148474
##
## lambda = 2.8 , RSS = 32.4593 , R2 = 0.615685
## lambda = 2.8 , RSS = 32.53004 , R2 = 0.6148474
##
## lambda = 2.9 , RSS = 32.46611 , R2 = 0.6156044
## lambda = 2.9 , RSS = 32.53004 , R2 = 0.6148474
##
## lambda = 3 , RSS = 32.47375 , R2 = 0.6155139
## lambda = 3 , RSS = 32.53004 , R2 = 0.6148474
##
## lambda = 3.1 , RSS = 32.48225 , R2 = 0.6154133
## lambda = 3.1 , RSS = 32.53004 , R2 = 0.6148474
##
```

```
## lambda = 3.2 , RSS = 32.49161 , R2 = 0.6153025
## lambda = 3.2 , RSS = 32.53004 , R2 = 0.6148474
##
## lambda = 3.3 , RSS = 32.50183 , R2 = 0.6151814
## lambda = 3.3 , RSS = 32.53004 , R2 = 0.6148474
##
## lambda = 3.4 , RSS = 32.51294 , R2 = 0.6150499
## lambda = 3.4 , RSS = 32.53004 , R2 = 0.6148474
##
## lambda = 3.5 , RSS = 32.52493 , R2 = 0.614908
## lambda = 3.5 , RSS = 32.53004 , R2 = 0.6148474
##
## lambda = 3.6 , RSS = 32.53781 , R2 = 0.6147555
## lambda = 3.6 , RSS = 32.53004 , R2 = 0.6148474
##
## lambda = 3.7 , RSS = 32.55158 , R2 = 0.6145924
## lambda = 3.7 , RSS = 32.53004 , R2 = 0.6148474
##
## lambda = 3.8 , RSS = 32.56625 , R2 = 0.6144187
## lambda = 3.8 , RSS = 32.53004 , R2 = 0.6148474
##
## lambda = 3.9 , RSS = 32.58183 , R2 = 0.6142342
## lambda = 3.9 , RSS = 32.53004 , R2 = 0.6148474
##
## lambda = 4 , RSS = 32.59832 , R2 = 0.614039
## lambda = 4 , RSS = 32.53004 , R2 = 0.6148474
##
```