

Eigenvalue and Eigenvector in R using Power Method

Rahul Goswami

2022-01-26

Power Method is a method for finding the eigenvalues and eigenvectors of a matrix. It is an iterative method.

Steps for the Power Method are:

Step 1: Initialize the vector v to be a random vector.

Step 2: Initialize the iteration counter to 0.

Step 3: Repeat the following until convergence:

Step 3.1: Calculate the matrix-vector product $A*v$.

Step 3.2: Normalize the vector v .

Step 3.3: Increment the iteration counter by 1.

Step 3.4: If the iteration counter is greater than 100, stop the iteration.

Step 3.5: If the iteration counter is less than 100, go to step 3.

Step 3.6: Return the eigenvector v . (Note: The eigenvector is not normalized.)

Let us consider a matrix A .

$$A = \begin{pmatrix} 1 & 2 & 0 \\ -2 & 1 & 2 \\ 1 & 3 & 1 \end{pmatrix}$$

Method

We will use power method, in which first of all we take a matrix with entry 1,1,1 as initial eigenvector then further multiply this with the matrix and we will get another 3×1 matrix then we will divide each element of the obtained matrix by maximum of that matrix, then we will repeat same till we get a reliable estimate, and we can further calculate dominant eigenvector by

$$\lambda = \frac{A \times x \cdot x}{x \cdot x}$$

Putting Matrix in R matrix function

```
A <- matrix(c(1,2,0,-2,1,2,1,3,1),ncol=3,byrow = TRUE)
vec <- matrix(c(1,1,1),ncol=1)
```

Initializing the iteration counter to 0

```
iteration <- 0
temp <- c(3,4,5)

while(any(temp != vec)){
  temp <- vec
  vec <- A %% vec
  vec <- round(vec/max(vec),4)
  iteration <- iteration + 1
  cat("Iteration:",iteration,"\t",vec,"\n")
}
```

```
## Iteration: 1      0.6 0.2 1
## Iteration: 2      0.4545 0.4545 1
## Iteration: 3      0.4839 0.5484 1
## Iteration: 4      0.5052 0.5051 1
## Iteration: 5      0.5017 0.4949 1
## Iteration: 6      0.4994 0.4994 1
## Iteration: 7      0.4998 0.5006 1
## Iteration: 8      0.5001 0.5001 1
## Iteration: 9      0.5 0.4999 1
## Iteration: 10     0.5 0.5 1
## Iteration: 11     0.5 0.5 1
cat("Dominant Eigenvector:",vec, "and Dominant Eigenvalue:",sum((A %*% vec) * vec)/sum( vec * vec))
## Dominant Eigenvector: 0.5 0.5 1 and Dominant Eigenvalue: 3
```