Practical 8

Calculate Eigenvalue and eigenvectors of following matrix using jacobi method

$$\begin{bmatrix} 1 & \sqrt{2} & 2 \\ \sqrt{2} & 3 & \sqrt{2} \\ 2 & \sqrt{2} & 1 \end{bmatrix}$$

workout

Putting the matrix in R

```
A<-matrix(c(1,sqrt(2),2,sqrt(2),3,sqrt(2),2,sqrt(2),1),nrow=3,byrow=T)
print(A)
```

```
##          [,1]     [,2]     [,3]
## [1,] 1.000000 1.414214 2.000000
## [2,] 1.414214 3.000000 1.414214
## [3,] 2.000000 1.414214 1.000000
```

Now we will use jacobi method to find different rotation matrix to , till our matrix off diagonal elements became zero

```
  i<-0                  #Just a counter variable
  l<-list()             # list to store J values
  repeat
  {
    Z<-A
    i<-i+1
    diag(A)<-0                                          #To consider only off diagonal element
    a<-as.vector(which(A==max(A),arr.ind = T)[2,])  #Extracting index of maximum of A
    B<-matrix(c(Z[a[1],a[1]],Z[a[1],a[2]],Z[a[2],a[1]],Z[a[2],a[2]]),nrow=2,byrow=T)
    theta<-(1/2)*atan((2*B[2,1])/(B[2,2]-B[1,1]))   #Calculating angle of rotation
    J<-diag(nrow(Z))                                    #defining rotation matrix
    J[a[1],a[1]]<-cos(theta)
    J[a[1],a[2]]<--sin(theta)
    J[a[2],a[1]]<-sin(theta)
    J[a[2],a[2]]<-cos(theta)
    A<-t(J) %*% Z %*% J
    A<-round(A,5)                                       #rounding upto decimal 5
    l[[i]]<-J
    if(all(A[row(A)!=col(A)]==0)) #checking if all off diagonal element is 0, stop iter.
    {
      break
    }
  }
  print(A)         #printing the matrix whose diagonal are eigenvalue
```

```
##      [,1] [,2] [,3]
## [1,]    5    0    0
## [2,]    0    1    0
## [3,]    0    0   -1
```

```
  print(l)        #list of rotation matrix
```

```
## [[1]]
##           [,1] [,2]       [,3]
## [1,] 0.7071068    0 -0.7071068
## [2,] 0.0000000    1  0.0000000
## [3,] 0.7071068    0  0.7071068
##
## [[2]]
##           [,1]       [,2] [,3]
## [1,] 0.7071068 -0.7071068    0
## [2,] 0.7071068  0.7071068    0
## [3,] 0.0000000  0.0000000    1
```

Now we have to store diagonal of the A in a vector, and multiply all the rotation matrix to get a matrix , lets call it b whose coloumns are eigencectors

```
eigenvalues<-diag(A)
  b<-l[[1]]
  for(i in 1:(length(l)-1))
  {
    b<-b%*%l[[i+1]]
  }
b<-round(b,3)
print(eigenvalues)
```

```
## [1]  5  1 -1
```

```
print(b)
```

```
##       [,1]   [,2]   [,3]
## [1,] 0.500 -0.500 -0.707
## [2,] 0.707  0.707  0.000
## [3,] 0.500 -0.500  0.707
```

Conclusion

Eigenvalue are 5, 1, -1 and corresponding eigenvectors are coloumns of

$$\begin{bmatrix} 0.5 & -0.5 & -0.707 \\ 0.707 & 0.707 & 0 \\ 0.5 & -0.5 & 0.707 \end{bmatrix}$$