

# Computing Integrals Monte Carlo Way

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## Introduction

Let us we have  $\theta$  with posterior density  $\pi(\theta|x)$ , then the posterior mean is given by

$$\mu(\theta) = \int_{-\infty}^{\infty} \pi(\theta|x) d\theta$$

And the posterior of a particular function  $h(\theta)$  is given by

$$E(h(\theta)|x) = \int_{-\infty}^{\infty} \pi(\theta|x) h(\theta) d\theta$$

Then through the Monte Carlo method we can estimate the posterior mean of  $E(h(\theta)|x)$  by taking  $N$  samples from the posterior distribution  $\pi(\theta|x)$ . And the posterior mean can be estimated by

$$\bar{h} = \frac{1}{N} \sum_{i=1}^N h(\theta_i)$$

## The Monte Carlo Method

Suppose we need to compute the integral

$$\int_0^1 12x^3(1-x)^2 dx$$

If we do integral manually we will get 0.2. But if we use the Monte Carlo method, we can get the same answer by taking  $N$  samples from the posterior distribution. If we look closely it's a beta distribution with parameter  $\alpha = 2$  and  $\beta = 3$ . and it is a second moment of the beta distribution. that mean  $h(x) = x^2$ . so we have to take  $N$  samples from the beta distribution with parameter  $\alpha = 2$  and  $\beta = 3$ . and square them. and at last take mean of them

```
sample = rbeta(n = 100000, 2, 3) # sample from beta distribution with parameters 2 and 3
sample_squared = sample^2
mean = mean(sample_squared)
standard_error = sd(sample_squared) / sqrt(100000)
cat("Estimate of the integral is ", mean, " with standard error ", standard_error, sep = "\n")

## Estimate of the integral is 0.199853 with standard error 0.0005601228
```