Finding Highest Posterior Density Interval

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Let us consider the following dataset follows an exponential distribution with scale parameter θ.Let us consider the prior for θ. Obtain posterior distribution, Bayes estimator, and 0.95 HPD interval for the parameter.

3.29, 7.53, 0.48, 2.03, 0.36, 0.07, 4.49, 1.05, 9.15,3.67, 2.22, 2.16, 4.06, 11.62, 8.26, 1.96, 9.13, 1.78, 3.81, 17.02

The density of the data model will be given by

$$f(x|\theta) = \frac{1}{\theta} e^{\frac{-x}{\theta}}$$

Let us notify $\sum_{i=1}^{n} x_i = S_n$ now the likelihood will be given by

$$L(x|\theta) = \left(\frac{1}{\theta}\right)^n e^{\frac{-S_n}{\theta}}$$

Now Since we do not have any info about θ let us assume non-informative prior

$$\pi(\theta) = \frac{1}{\theta}$$

Then the posterior will be given by

$$\pi(\theta|x) = \frac{\frac{1}{\theta} \cdot \left(\frac{1}{\theta}\right)^n e^{\frac{-S_n}{\theta}}}{\int_0^\infty \frac{1}{\theta} \cdot \left(\frac{1}{\theta}\right)^n e^{\frac{-S_n}{\theta}}}$$

$$\pi(\theta|x) = \frac{S_n^n}{\Gamma(n)} \cdot \left(\frac{1}{\theta}\right)^{n+1} e^{\frac{-S_n}{\theta}}$$

Now this is the density of the Inverse Gamma so

$$\pi(\theta|x) \sim Inv - Gamma(n, S_n)$$

So the bayes estimate will be given by $\frac{S_n}{n-1}$

Code

```
xobs <- c(3.29, 7.53, 0.48, 2.03, 0.36, 0.07, 4.49, 1.05, 9.15,3.67, 2.22, 2.16, 4.06, 11.62, 8.26, 1.9
Bayes_Estimate = sum(xobs)/(length(xobs)-1)
cat("Bayes Estimate of scale parameter is given by ",Bayes_Estimate)
```

```
## Bayes Estimate of scale parameter is given by  4.954737
```

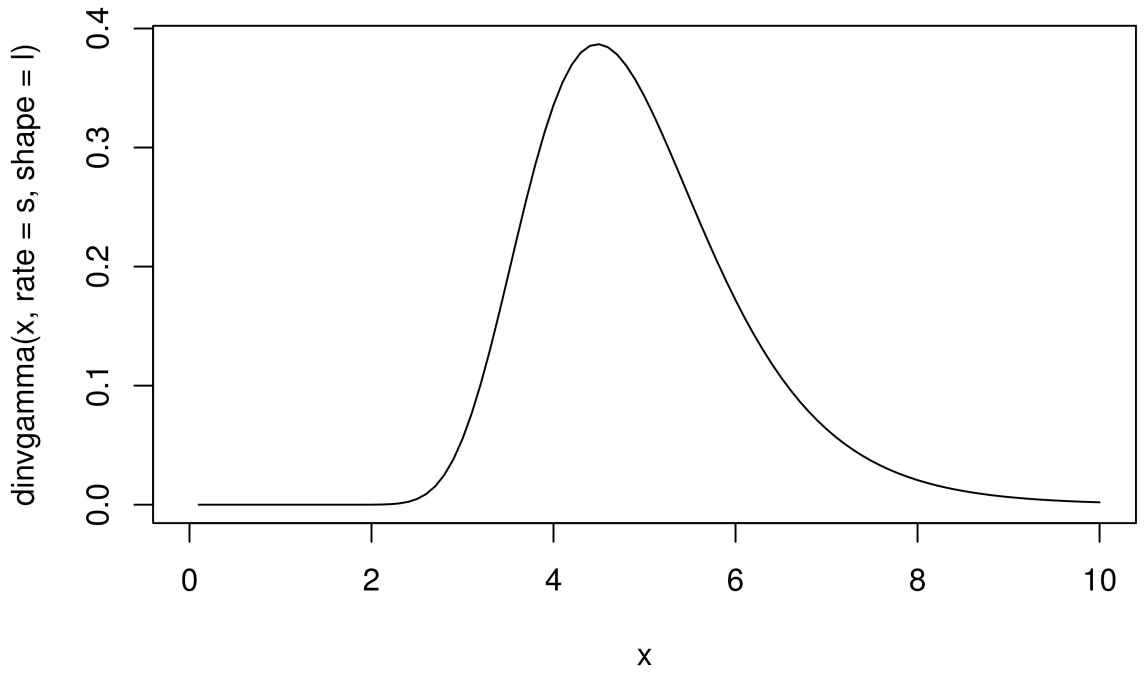
Now **HPDI** will br given by

$$\int_{\theta:\pi(\theta|X)\geq k} \pi(\theta|X)d\theta = 1-\alpha$$

where $1-\alpha = 0.95$, here it can be thought as a horizontal line is on the posterior density such that the point where the posterior density intersect this line the area between these points will be 0.95

Let us take a look at posterior density function

```
s = sum(xobs)
l =length(xobs)
curve(dinvgamma(x , rate = s , shape = l),from=0,to=10)
```



Now let us find HPD **Code**

```
ruler1 <- seq(2, s/(l+1),length=3500 )  #s\(l+1) is mode of posterior
ruler2 <- seq(s/(l+1), 8 ,length = 5000)
target = 0.95
tolerance = 0.0005
done<- FALSE
for(i in ruler1)
{
  for(j in ruler2)
  {
    if(round(dinvgamma(i,rate=s,shape = l),3)==round(dinvgamma(j,rate=s,shape = l),3))
    {
      #print(paste(i,"and",j))
```

```
      L <- pinvgamma(i,rate=s,shape=l)
      H <- pinvgamma(j,rate=s,shape=l)
      if (((H-L)<(target+tolerance)) & ((H-L)>(target-tolerance)))
      {
        done <- TRUE
        break
      }
    }
  }
 if (done){break}
}
HPD.L <- i; HPD.U <- j
print(paste(target*100, "% HPD interval:", HPD.L, "to", HPD.U))
```

```
## [1] "95 % HPD interval: 2.94588413015964 to 7.2851736061498"
```